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Letter

Detecting zero-point fluctuations with stochastic Brownian oscillators

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High-quality quantum oscillators are preferred for precision sensing of external physical parameters because if the noise level due to interactions with the environment is too high, metrological information can be lost due to quantum decoherence. On the other hand, stronger interactions with a thermal environment could be seen as a resource for new types of metrological schemes. We present a general amplification strategy that enables the detection of zero-point fluctuations using low-quality quantum oscillators at finite temperature. We show that by injecting a controllable level of multiplicative frequency noise in a Brownian oscillator, quantum deviations from the virial theorem can be amplified by a parameter proportional to the strength of the frequency noise at constant temperature. As an application, we suggest a scheme in which the virial ratio is used as a witness of the quantum fluctuations of an unknown thermal bath, either by measuring the oscillator energy or the heat current flowing into an ancilla bath. Our work expands the metrological capacity of low-quality oscillators and can enable new measurements of the quantum properties of thermal environments by sensing their zero-point contributions to system variables.

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Introduction. Brownian motion of classical and quantum oscillators subject to dissipation and noise fluctuations is a paradigmatic model with direct fundamental [1–4] and experimental [5,6] implications in several areas of physics, from state tomography and spectroscopy [7,8] to tests on gravitation [9]. Dissipation and noise result from the interaction of the system oscillator with its environment, often assumed as a large set of degrees of freedom that are not controlled but influence the dynamics of the system [10]. Strong interactions with a thermal environment are avoided in metrological schemes, as they reduce the sensitivity of system properties to external parameters such as external fields. This is critical in quantum sensing, as the metrological information content is severely limited by environment-induced decoherence [11,12].

Implementing high-quality (high-Q) oscillators that are well isolated from the environment is essential for metrological schemes that map unknown electric and magnetic fields to variations in the oscillator frequency [13]. Frequency shifts are more difficult to measure with noisier low-Q oscillators. On the other hand, stronger dependence of the oscillator observables on the detailed structure of the environment can open new opportunities for obtaining information from the surroundings. For a system interacting with an environment formed by a mediator field coupled to a source system, studying the system's effective dynamics could reveal fundamental aspects of the mediator field even without having access to its full description. This might be of great interest for deciding, for instance, whether or not gravity is quantum. Quantum mechanical proposals to address this problem include full spectroscopy of the environment [14], generation of entanglement through a mediator field [9,15], or studying decoherence due to environmental fields [16,17]. However, witnessing the nature of an environment might be accessible without appealing to intrinsic quantum features (entanglement, decoherence), as it occurs with Casimir forces [18,19], where zero-point fluctuations manifest macroscopically [20].

For classical harmonic oscillators that are completely isolated from their environments, the virial theorem establishes that the average kinetic energy $\langle K\rangle$ equals the average potential energy $\langle V\rangle$ [21]. The virial ratio $\mathcal{R} \equiv \langle K\rangle/\langle V\rangle$ is also equal to one for quantum harmonic oscillators that undergo unitary dynamics [22]. Environment-induced deviations of $\mathcal{R}$ from unity have only recently been explored in the quantum regime from a formal perspective [23,24], but a connection between the virial ratio and experimentally accessible oscillator variables has not been developed yet.

In this Letter we show that a low-Q Brownian oscillator that is subject to a frequency noise at low temperatures can be used to probe environmentally induced changes in the virial ratio $\mathcal{R}$, either by measuring the oscillator energy E or heat currents J between the system and the environment. Deviations from $\mathcal{R} = 1$ are shown to scale with the tunable magnification factor $\mathcal{W} = QD\Omega/(1 - QD\Omega)$, where D is the frequency noise strength and Ω the oscillator natural frequency. Such a magnification can be exploited for precision measurements. At low temperatures, the proposed scheme is shown to represent a feasible approach for measuring zero-point fluctuations. We also propose a two-bath protocol for probing the quantum nature of fluctuations of an unknown thermal bath that couples to the frequency-driven Brownian oscillator, by measuring the oscillator energy or the heat current flow between the system and an ancilla bath, as illustrated in Fig. 1.

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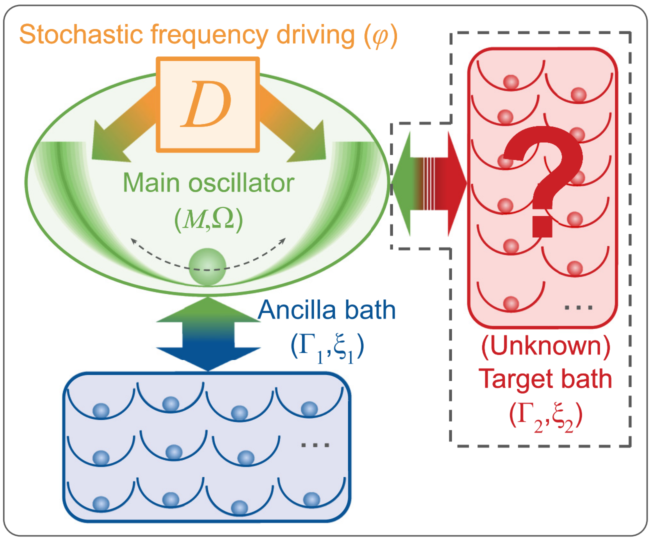

FIG. 1. Scheme of the scenario. A nonequilibrium Brownian oscillator of mass M and frequency Ω is subjected to white frequency noise (given by the stochastic variable φ and characterized by the strength D). The oscillator is coupled to two Ohmic baths characterized by damping constants Γ_1 and Γ_2, stochastic force $\xi_{1,2}$, and a large cutoff frequency ($\Omega_C \gg \{\Omega, \Gamma_{1,2}\}$). An Ancilla bath is assumed to introduce strong dissipation ($\Gamma_1 \lesssim \Omega$). The nature of the target bath is assumed unknown.

Stochastic Brownian oscillator. We consider a harmonic oscillator whose frequency is randomly driven and simultaneously interacts with one or more thermal environments. Frequency noise is given by $\Omega(t) = \Omega[1+\varphi(t)]^{1/2}$, with white noise amplitude $\varphi(t)$ of the zero mean and second moment $\langle\varphi(t)\varphi(t')\rangle = 2D\delta(t-t')$. D is the noise strength (in Hz^{-1}). Higher-order cumulants are $\langle\langle\varphi(t_1)\cdots\varphi(t_n)\rangle\rangle = 2^n D_n\delta(t_1-t_2)\cdots\delta(t_{n-1}-t_n)$ with D_n taken as in Ref. [25]. Physically, frequency fluctuations could be implemented using laser intensity noise in levitated nanoparticles by optical tweezers [26], voltage noise in an ion trap [7], or spectral diffusion in single-molecule substrates [27] (see Sec. A of Supplemental Material (SM) for suggested physical implementations [28]). We neglect any nonlinear backreaction associated with the injection of noise energy in the oscillator [29]. For the analysis below, the oscillator can be classical or quantum mechanical.

Thermal environments correspond to sets of harmonic oscillators in thermal states with temperature T_k, where k labels different baths ($k = \{1, 2\}$ in Fig. 1). Bath oscillators are linearly coupled to the main system oscillator (see the Caldeira-Leggett model details in SM Sec. B [28]). Eliminating the bath variables results in damping terms and stochastic fluctuation forces represented by the random variables ξ_k, acting on the main oscillator. The former are given by $(d/dt)[\int_0^t d\tau\tilde{\Gamma}_k(t-\tau)x(\tau)]$, having $x(t)$ as the main oscillator amplitude and $\tilde{\Gamma}_k(t)$ the damping kernels which depend on the bath cutoff frequency Ω_C and the damping constants Γ_k.

The system dynamics is non-Markovian in general, but for large cutoff frequencies $\Omega_C \gg \{\Omega, \Gamma_{1,2}\}$ we have $(d/dt)[\int_0^t d\tau\tilde{\Gamma}_k(t-\tau)x(\tau)] \approx 4\Gamma_k\dot{x}(t)$, which gives Markovian evolution. The resulting equation of motion for $x(t)$ reads

$$\ddot{x}(t) + \Omega^2[1+\varphi(t)]x(t) + 4\Gamma\dot{x}(t) = \frac{\xi(t)}{M}, \tag{1}$$

where $\Gamma = \Gamma_1 + \Gamma_2$ and $\xi = \xi_1 + \xi_2$, with stochastic force correlations given by $\langle\xi(t)\xi(t')\rangle = (1/2)[N_1(t-t') + N_2(t-t')]$ due to the independence of the baths [i.e., $\langle\xi_1(t)\xi_2(t')\rangle = 0$]. The noise kernels (symmetrized correlations) are $N_k(t-t') = \langle\xi_k(t)\xi_k(t') + \xi_k(t')\xi_k(t)\rangle$, where the expectation value is over the bath ensemble. For quantum thermal baths, noise kernels are given by ($\hbar = 1$) [30,31],

$$N_k(t-t') = 2\int_0^{+\infty} d\omega\mathcal{J}_k(\omega)\coth\left(\frac{\omega}{2k_B T_k}\right)\cos[\omega(t-t')], \tag{2}$$

where $\mathcal{J}_k(\omega) = (2M\Gamma_k\omega/\pi)f(\omega/\Omega_C)$ is the spectral density of the kth bath, which we assume to be Ohmic with a Lorentzian cutoff function $f(x) = 1/(1+x^2)$. The thermal factor $\coth(\omega/[2k_B T_k]) = 1 + 2\bar{n}_k(\omega)$ presents the sum of the contributions of the zero-point and the thermal fluctuations, with the latter associated to the Bose-Einstein distribution $\bar{n}$. The high-temperature limit for a given bath corresponds to $\Omega, \Gamma_{1,2} \ll \Omega_C \ll k_B T_k$, which allows the approximation $N_k(t-t') \approx 4M\Gamma_k k_B T_k\Omega_C\exp[-\Omega_C(t-t')]$, i.e., exponentially correlated noise. A sufficiently large cutoff frequency gives the classical $N_k(t-t') \to 4M\Gamma_k k_B T_k\delta(t-t')$, corresponding to white noise. Comparing the latter form with the damping kernel in the large cutoff limit, the classical Fluctuation-Dissipation Relation (FDR) is verified. In this Letter, we consider the high-temperature limit as a definition of classical baths. This criteria relies on the fact that the classical limit neglects both the zero-point fluctuations and the (quantum) blackbody features of the occupation number $\bar{n}_k$.

Steady-state energy distribution. Equation (1) without dissipation describes parametric resonances when the variable $\varphi(t)$ is replaced by harmonic driving ($\varphi(t) \to A_D\cos[\omega_D t]$), with some amplitude A_D and driving frequency Ω_D. For specific values of ω_D, the oscillator undergoes exponential growth of $x(t)$ with time [32]. In our case, frequency driving is not harmonic but random, and white noise injects energy on the system at all frequencies. A sufficient amount of dissipation can compensate for this energy injection and lead the system to a stable steady state. Unbalanced energy injection can give unstable dynamics. Stability criteria for our case are discussed below.

In the large cutoff regime, we follow the approach in Ref. [25] for solving Eq. (1) to obtain first- and second-order moments of x. Analytical expressions in the steady state can be derived under the assumption of white noise for φ, while preserving the quantum fluctuations on the bath, despite the main oscillator being formally treated as statistically classical (see Refs. [33,34] for approaches restricted to the classical case). From $\langle x^2\rangle$ and $\langle p^2\rangle$, the stationary energy E of the frequency-driven oscillator can be written as

$$E = E_0[1 + 2\mathcal{W}/(1+\mathcal{R})], \tag{3}$$

where $E_0 = \langle K\rangle + \langle V\rangle$ and $\mathcal{R}$ are the energy and the virial ratio of the undriven Brownian oscillator ($D = 0$), while $\mathcal{W} = QD\Omega/(1-QD\Omega)$ is the amplification factor, with $Q = \Omega/4\Gamma$

the quality factor of the main oscillator. The derivation of Eq. (3) is summarized in SM Sec. C [28].

The dynamical stability of the oscillator matches the positivity of the energy. The latter is given by the condition $QD\Omega < 1$, encoding the competence between the dissipation and the strength of the frequency noise (see, for instance, Ref. [35]). Within the space of stable solutions, the fact that the correction to the undriven energy E_0 scales with D, for fixed Q and Ω, opens the opportunity of probing the environment induced deviations of the classical virial ratio $\mathcal{R} = 1$. Such deviations have been discussed [24], but feasible experimental schemes for detecting such deviations are not available. Equation (3) shows that by directly measuring the energy of a classical or quantum Brownian oscillator with tunable frequency noise strength D, bath-induced deviations from the virial theorem can be amplified in stationary measurements. Rewriting Eq. (3) as $E/E_0 = 1 + \mathcal{F}\mathcal{W}$, with $\mathcal{F} = 2/(1+\mathcal{R})$, suggests that by measuring E with different driving strengths $D \sim \mathcal{W}$, deviations from the classical virial ratio could be found from the slope $\mathcal{F} \leqslant 1$, with the equality holding in the classical limit $\mathcal{R}_{\rm cl} = 1$. Also, it is worth noting that other forms of writing the energy open the possibility for measuring the steady state value of $\langle x^2\rangle$ at $D = 0$ with a similar strategy as for $\mathcal{R}$ (see SM Sec. C [28]).

Detecting deviations from the virial theorem. To understand the regimes of $\mathcal{R}$ that could accessible with oscillator energy measurements, it is instructive to rewrite the stationary solution for $\langle K\rangle$ as (see SM Sec. C [28] for the derivation)

$$\langle K\rangle = \frac{\Omega}{2}\int_0^\infty \frac{du}{\pi}\frac{u\coth(u/\widetilde{T})}{Q([1-u^2]^2+[u/Q]^2)}u^2 f(u/u_{\rm C}), \quad (4)$$

where $\widetilde{T} = T/T_0$ is the ratio between the classical thermal energy ($E_0^{\rm (cl)} = k_{\rm B}T$) and the oscillator zero-point energy ($\Omega/2$), i.e., $T_0 \equiv \Omega/(2k_{\rm B})$, and $u_{\rm C} \equiv \Omega_{\rm C}/\Omega$ is a cutoff parameter. The potential energy $\langle V\rangle$ has the same integral expression as $\langle K\rangle$ with the replacement $u^2 f(u/u_{\rm C}) \to 1$ (see SM Sec. C [28]). Only the kinetic energy needs to be regularized with a cutoff function mantaining the well-known dependence on $u_{\rm C}$ [36]. Deviations from the classical virial theorem ($\mathcal{R}_{\rm cl} = 1$) can be expected in the combined regime of low temperatures ($\widetilde{T} \lesssim 1$) and strong system-bath coupling (low Q). For high-temperatures [$\widetilde{T} \gg u_{\rm C}$ in Eq. (4)] the classical ratio $\langle K\rangle = \langle V\rangle \approx k_{\rm B}T/2$ holds for all system-bath coupling strengths, in agreement with the equipartition theorem. For high-Q oscillators (equivalent to the weak-coupling limit, $Q \gg 1$), we have that $u/[\pi Q([1-u^2]^2 + u^2/Q^2)] \to [1/(2u)][\delta(u-1)+\delta(u+1)]$, giving $\langle K\rangle = \langle V\rangle = (\Omega/4)\coth(1/\widetilde{T})$, satisfying $\mathcal{R} = 1$ at all temperatures (see SM Sec. C [28]).

Figure 2 shows the virial ratio $\mathcal{R}$ for a Brownian oscillator as a function of the reduced temperature $\widetilde{T}$, for different quality factors Q. The cutoff parameter is set to $u_{\rm C} = 10^3$. As discussed above, the classical limit $\mathcal{R}_{\rm cl} = 1$ is approached as the thermal energy exceeds the zero-point energy ($\widetilde{T} \gg 1$) and low-temperature deviations from the classical limit are suppressed as the coupling to the bath becomes weaker (higher Q values). Up to ~30% deviations from the classical limit are expected at lower temperatures ($\widetilde{T} \sim 0.1$) for relatively lossy oscillators ($Q \sim 10$). We ruled out that the calculated deviations from $\mathcal{R}_{\rm cl} = 1$ are a cutoff-dependent artifact by comparing with the fundamental lower bound $\mathcal{R} \geqslant \mathcal{R}_{\rm H} \equiv \Omega^2/16\langle V\rangle^2$, obtained from Heisenberg's uncertainty relation $\Delta p^2\Delta x^2 \geqslant 1/4$, given that $\langle x\rangle = \langle p\rangle = 0$. Since $\mathcal{R}_{\rm H}$ only depends on the potential energy, it is effectively cutoff independent in the large cutoff regime. While the Brownian oscillator model gives deviations from the virial theorem that depend on the cutoff parameter, the Heisenberg bound guarantees that there are bath-induced deviations at very low temperatures.

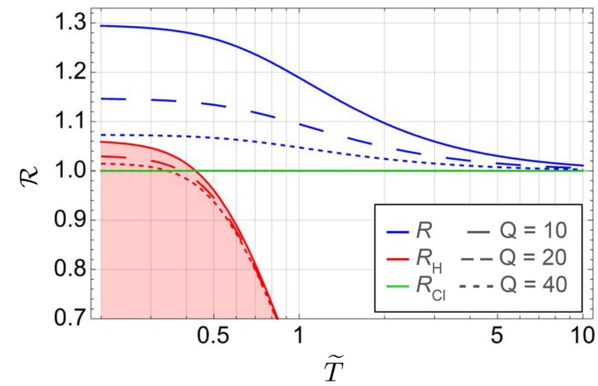

FIG. 2. Virial ratio $\mathcal{R} = \langle K\rangle/\langle V\rangle$ as a function of the normalized temperature $\widetilde{T}$ for an oscillator with quality factor $Q = 10, 20, 40$ (solid, long dashed, short dashed) and a cutoff frequency for the bath $Q_{\rm C} = 10^3$. The blue curves correspond to the full expression for a Brownian oscillator in the steady state under the influence of a quantum thermal bath, including zero-point fluctuations. Red curves correspond to the lower bound on $\mathcal{R}$ obtained from applying the Heisenberg uncertainty principle (shaded area). The green solid line corresponds to the value of the ratio for a Brownian oscillator under the influence of a "classical" thermal bath.

As mentioned above, there is a limit to how strongly the oscillator can be driven without undergoing parametric amplification into unstable steady states. Figure 3(a) shows the magnitude of the net energy deviation $\Delta = E^{\rm (cl)}/E_0^{\rm (cl)} - E/E_0 = \mathcal{W}(1-\mathcal{F})$ when the bath is either classical or quantum, expected for different Q factors and dimensionless noise strength $D\Omega$, for stable configurations satisfying $QD\Omega < 1$. Relatively large net energy deviations $\Delta \sim 0.2$ are expected for $Q < 10$ and φ-noise strength $\sqrt{2D\Omega} \sim 0.35$. This suggests that low-Q oscillators are more suitable for observing larger variations of $\mathcal{F}$.

Deviations from classical virial theorem are larger at lower temperatures, because zero-point bath fluctuations become important. A manifestation of this is the strong dependence of $\mathcal{R}$ with the bath's cutoff frequency $\Omega_{\rm C}$. This dependence is lost in the high-temperature (classical) limit when the Bose-Einstein distribution cutoff $\overline{n}$ dominates over $\Omega_{\rm C}$. In other words, by measuring the total oscillator energy E, and accessing $\mathcal{R}$, it is possible to assess whether the thermal environment is quantum or classical. Figure 3(b) shows the differences in slopes $\Delta\mathcal{F}_1 = |\mathcal{F}_{\rm Q} - \mathcal{F}_{\rm cl}|$ between quantum and classical thermal baths, as a function of the dimensionless temperature $\widetilde{T}$. Quantum baths satisfy FDR via the quantum thermal factor $\coth(u/\widetilde{T})$, i.e., including both zero-point and blackbody contributions, and classical baths satisfy FDR through the high-temperature factor $\widetilde{T}/u$. For both cases, we set $Q = 10$

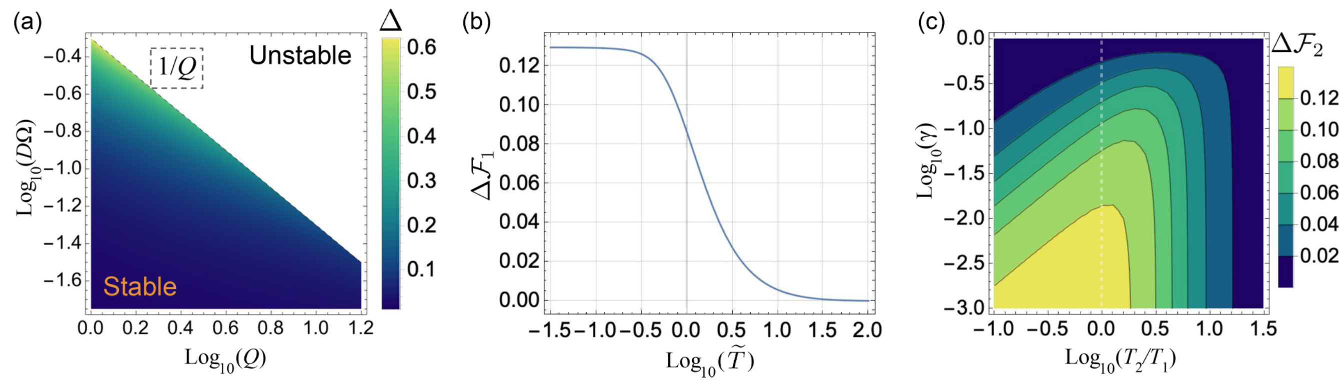

FIG. 3. (a) Net energy deviation $\Delta = E^{(\mathrm{cl})}/E_0^{(\mathrm{cl})} - E/E_0 = \mathcal{W}(1-\mathcal{F})$ over the stability region of an oscillator at temperature $\widetilde{T} = 1/4$ as a function of $D\Omega$ and Q, with $u_\mathrm{C} = 10^3$. (b) Difference of the virial factor $\Delta\mathcal{F}_1 = |\mathcal{F}_\mathrm{Q} - \mathcal{F}_\mathrm{cl}|$ as a function of the normalized temperature $\widetilde{T}$ for the single bath case when the bath is either quantum or classical, with $Q = 10$ and $u_\mathrm{C} = 10^3$. (c) Variation of the virial factor $\Delta\mathcal{F}_2 = |\mathcal{F}_{2,\mathrm{Q}} - \mathcal{F}_{2,\mathrm{cl}}|$ for the two-bath scenario with temperatures $T_{1,2}$ and different damping rates such that the relative damping is defined $\gamma \equiv \Gamma_1/(\Gamma_1+\Gamma_2)$. The variation is considered between scenarios where the ancilla bath is fixed to be quantum and the target is either quantum or classical. The dashed vertical line corresponds to thermal equilibrium ($T_1 = T_2$), with $\widetilde{T}_1 = 1/4$, $Q = 10$, and $u_\mathrm{C} = 10^3$.

and $u_\mathrm{C} = 10^3$. As expected, at higher temperatures zero-point contributions cannot be distinguished, but differences $\Delta\mathcal{F}$ of over 12% are predicted at low temperatures.

Probing the nature of an unknown bath. Stochastic frequency Brownian oscillators can be used as probes of a second bath (see Fig. 1). A known *ancilla* bath with tunable damping rate Γ_1 and temperature T_1 is coupled to the main oscillator, which in turn is coupled to a second *target* bath, such that only the damping constant Γ_2 is previously known from the oscillator's relaxation dynamics without frequency noise. The temperature of the target bath (T_2) is only approximately estimated, so it is possible to look for differences in the virial ratio. The ancilla bath is quantum mechanical (i.e., includes zero-point fluctuations and Bose-Einstein thermal fluctuations), but the quantum or classical nature of the target bath is unknown.

For nonequilibrium scenarios ($T_1 \neq T_2$), a steady heat current is expected to flow between baths from higher to lower temperature, through the main oscillator [2]. Frequency noise acts as an additional energy injection source, therefore contributing to the stationary heat current between baths. From the power balance expression $dE/dt = J_1 + J_2 + W$, where $W \equiv \Omega^2\langle\varphi x p\rangle$ is the energy injected by the frequency noise, we extend the methods in Refs. [25,37] to obtain the heat current J_k from the driven oscillator to the kth bath of the form (derivation in SM Sec. D [28])

$$J_k = J_k^{(0)} - 4\Gamma_k E_0 \mathcal{W}\mathcal{F}, \tag{5}$$

where the first term $J_k^{(0)}(T_1, T_2)$ is the standard heat current without frequency noise, and the second term is a noise-induced correction, which again scales linearly with the magnification factor $\mathcal{W}$ and the virial factor $\mathcal{F}$. The virial factor is a function of the properties of the two baths, i.e., $\mathcal{F} = \mathcal{F}(T_1, T_2)$. The same occurs for the energy $E_0 = E_0(T_1, T_2)$. The quality factor Q is defined over the total damping $\Gamma = \Gamma_1 + \Gamma_2$.

Equation (5) suggests that measurements of current J_1 into the ancilla bath can be used to determine the nature of the target bath, by detecting the value of $\mathcal{F}$ for a specific scenario. To illustrate this point, a measurement protocol can be proposed: (i) Ω and Γ_1 are obtained from a measurement of the decay of the oscillator only coupled to the ancilla bath and $D = 0$. (ii) Γ_2 is obtained from a similar measurement as before when the oscillator is coupled to both baths. (iii) Measurements on the oscillator when $D = 0$ give E_0 and $J_1^{(0)}$. (iv) Energy or ancilla current measurements are performed for different values of D (thus $\mathcal{W}$), obtaining from the slope the value of $\mathcal{F}$.

Figure 3(c) shows the variation $\Delta\mathcal{F}_2 \equiv |\mathcal{F}_{2,\mathrm{Q}} - \mathcal{F}_{2,\mathrm{cl}}|$ defined as the difference in $\mathcal{F}$ when the ancilla bath is set to be quantum but the target is either quantum or classical, as a function of the fractional ancilla damping parameter $\gamma \equiv \Gamma_1/(\Gamma_1+\Gamma_2)$ and the temperature ratio T_2/T_1 (log scale). We set $Q = 10$, $Q_\mathrm{C} = 10^3$, and $\widetilde{T}_1 = 1/4$. The 12% deviation in Fig. 3(b) stands as the maximum difference, occurring for small γ (i.e., negligible contributions of the ancilla bath) and low temperatures of the target bath. As thermal differences increase, one of the two baths dominates over the other. If the target bath is the coldest, the differences are larger. Considering the dependence with γ, the difference increases when the target bath is dominant. However, decreasing γ implies a smaller J_1, suggesting an interplay between having a dominant target bath and a measurable J_1. Differences up to 2% are found over broad nonequilibrium conditions when the ancilla bath has the smallest damping rate.

Conclusion. We have shown that a stationary Brownian oscillator subject to frequency (multiplicative) noise can be used to assess the quantum or classical nature of a target bath, based on deviations from the classical virial ratio $\langle K\rangle/\langle V\rangle = 1$, which can be magnified by a suitabe tuning of the noise strength D. This is achieved by either measuring the total oscillator energy E or the heat current flowing to an ancilla bath, but without accessing or intervening directly the target bath by means of direct measurements on it or sensing its interaction with the oscillator. We show that due to stability constraints, lower-quality oscillators ($Q < 10$) subject to strong frequency driving are best suited for detecting deviations from the virial theorem, which are shown to exceed 10% for thermal energies below the zero-point motion of the driven oscillator ($2k_BT <$

Ω). Since frequency noise admits potential implementations in a variety of available physical platforms such as levitated nanoparticles in fluctuating optical tweezers [26], trapped ions in fluctuating Paul traps [7], optical cavities with fluctuating walls (see SM Sec. A [28]), and single molecules undergoing spectral diffusion [27], our theoretical predictions can be readily tested.

Generalizations of our results to oscillators with different spectral densities for the baths or colored frequency noise are possible. Different physical implementations could be relevant in hybrid quantum platforms such as an ancilla bath being electromagnetic and the target bath being gravitational [38], phononic, or plasmonic [39], although case by case studies are required depending on the implementation. Our work thus opens avenues for sensing the fundamental nature of an environment, with perspectives towards other applications that might leverage from the magnification factor for increasing the measurement's sensitivity.

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Data availability. The data supporting this study's findings are available within the article.

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